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INVESTIGATING THE EFFECT OF ROTATIONAL DEGREE OF FREEDOM ON A CIRCULAR CYLINDER AT LOW REYNOLDS NUMBER IN CROSS FLOW

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Summary. Numerical simulations of Vortex-Induced Vibrations (VIV) of a circular cylinder in cross flow with a rotational degree of freedom about its axis have been carried out by means of a finite-volume method. The study is performed in two dimensions at a Reynolds number of $Re_D = 100$, based on the free stream velocity and the diameter, D, of the cylinder. The effect of the rotational degree of freedom on the cylinder's lift and drag forces are compared with the baseline simulation results of flow around a stationary cylinder. The introduction of a rotational degree of freedom (d.o.f) is observed to cause the lift and drag forces to change. Also, the pattern of vortex shedding behind the cylinder is found to drastically change when the cylinder is allowed to rotate.

1. INTRODUCTION

The study of flows around cylinders has a long history [1, 2]. The early studies were focussed on the flow around a stationary cylinder at various Reynolds numbers. Subsequently, investigations have been carried out of flow around a cylinder with a prescribed rotational velocity [2]. Although the study of flows around rotating circular cylinders is not new, most of the previous works consider the rotational speed as a parameter that can be used to decrease the effect of vortex shedding on the cylinder. In other words, angular velocity is viewed as a way to reduce the root mean square of the lift force. This is the reason why rotation of the cylinder is used in feedback control of wakes [3]. Forced oscillatory rotation of a circular cylinder has also been investigated numerically as well as experimentally [4]. All of these studies focused on imposed oscillatory angular velocities. Etienne and Fontaine [5] studied the effect of vortex shedding on a two dimensional cylinder with two spatial d.o.f. They observed that the cylinder was mainly oscillating transversely and slightly in line with the flow. When they added a rotational degree of freedom, for an arbitrary rotational moment of inertia, the transverse amplitude of oscillation was found to be reduced by a factor of two, while the mean in-line deflection was also found to decrease by a factor between1.5 to 2. In their case, the Magnus effect was found to be negligible as the maximum angular velocity was only on the order of 5% of the free-stream velocity U [5].

In this study, we evaluate the effect of introducing a rotational degree of freedom, on the flow around a circular cylinder. To achieve this we introduce the rotational angle of the cylinder as an unknown that is affected by friction-induced torque.

Below, we present some of the results obtained by performing a parametric study in which the moment of inertia (I) and the rotational spring rigidity (k) are varied. The spring rigidity is used to control the rotational degree of freedom (d.o.f) and both k and I determine the natural frequency (f) of the system. An important parameter in this context is the so-called reduced velocity, Ur = U/fD, where U is the free-stream velocity and D is diameter of the cylinder. It should be noted that both k and I are defined for a unit-length cylinder.

2. GENERAL SPECIFICATION

The cylinder was allowed to rotate about its axis. The rotation was controlled by adding a torsional spring with stiffness K. Without the presence of the spring the cylinder was observed to rotate rigidly in one direction.

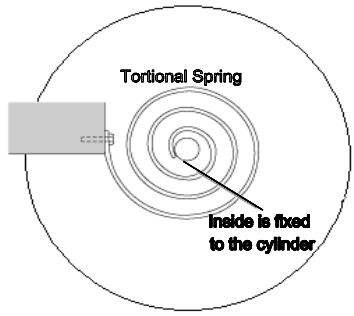
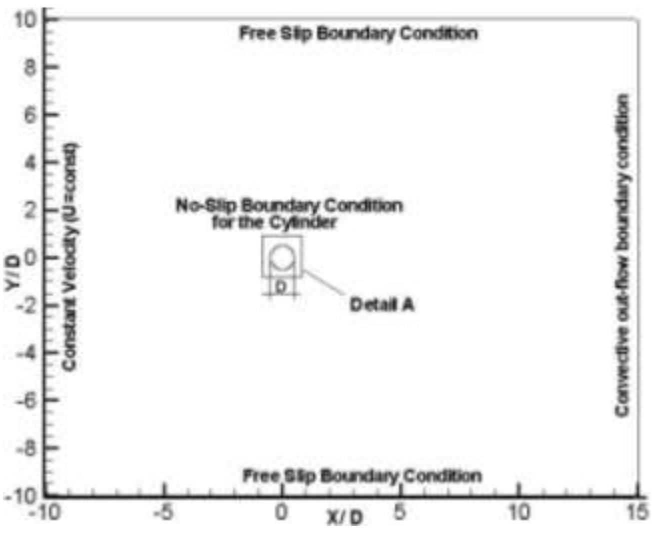


Figure 1 : Cylinder with Rotational degree of freedom, a Tortional spring- stiffness of K and the moment of Inersial of I

Because of the simplicity of the problem and the low Reynolds number, we were able to model the set up as a two-dimensional flow problem. The computational domain is shown in figure 2. At the inlet, the flow is assumed to be uniform with $u=U_0$ and v=0, where u and v are the velocities of the flow in x-direction and y-direction respectively. A free-slip boundary condition is applied along the upper and lower boundaries while a convective outflow boundary condition is applied at the outlet. At the surface of the cylinder, finally, a no-slip boundary condition is prescribed.



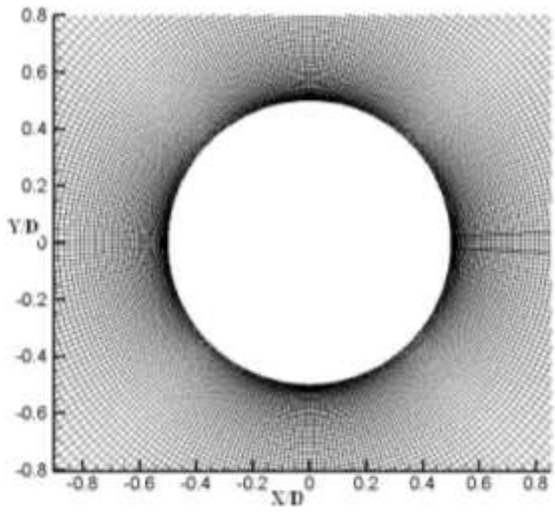


Figure 2: Left: The computational domain showing the boundary conditions. Right: Zoomed view of the O-mesh close to the cylinder corresponding to “Detail A” at the left

This problem is an example of a rotational harmonic oscillator that can oscillate about the axis of the cylinder. This behaviour is analogous to linear spring-mass oscillators. The general equation of motion is given by:

$$I\frac{d^2\theta}{dt^2} + C\frac{d\theta}{dt} + \kappa\theta = \tau(t) \qquad \textbf{(1)}$$

If the damping is small, $C \ll \sqrt{\kappa I}$, as is the case in this study, the frequency of vibration is very close to the natural resonance frequency of the system:

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\kappa/I} \qquad \textbf{(2)}$$

In the absence of a driving force ($\tau = 0$), the general solution of the resulting homogeneous problem is given by:

$$\theta = Ae^{-\alpha t}\cos(\omega t + \phi) \qquad \textbf{(3)}$$

Where:

$$\alpha = C/2I$$
$$\omega = \sqrt{\omega_n^2 - \alpha^2} = \sqrt{\kappa/I - (C/2I)^2} \qquad \textbf{(4)}$$

Table 1 : Definition of terms in the equations

Definition of terms		
Term	**Unit**	**Definition**
θ	Radians	Angle of deflection from rest position
I	$\mathrm{kg\,m^2}$	Moment of inertia
C	$\mathrm{kg\,m^2\,s^{-1}\,rad^{-1}}$	Rotational friction (damping)
κ	$\mathrm{N\,m\,rad^{-1}}$	Coefficient of torsion spring
τ	$\mathrm{N\,m}$	Drive torque
f_n	Hz	Undamped (or natural) resonance frequency
ω_n	$\mathrm{rad\,s^{-1}}$	Undamped resonance frequency in radians
f	Hz	Damped resonance frequency
ω	$\mathrm{rad\,s^{-1}}$	Damped resonance frequency in radians
α	$\mathrm{s^{-1}}$	Reciprocal of damping time constant
ϕ	Rad	Phase angle of oscillation
L	M	Distance from axis to where force is applied

The angular velocity of the cylinder is determined by a numerical approximation of eq. (1) with C=0, using an Euler scheme for the integration of time. The torque τ(t) is calculated every time step by integrating the tangential frictional forces of the flow on the cylinder.

3. NUMERICAL RESULTS

For this simulation the LESOCC flow solver has been used. LESOCC has been developed at the Institute of Hydromechanics at Karlsruhe, Institute of Technology, Germany. In Wissink and Rodi [6] it has been extensively tested for the simulation of flow around a cylinder at Re=3200. LESOCC uses a second-order accurate discretization of the convection and diffusion, combined with a three-stage Runge-Kutta method for the time-integration. It uses a collocated variable arrangement combined with momentum interpolation to avoid a decoupling of the pressure and velocity fields.

For the present study, a mesh independency test was carried out and, as a result, a mesh with (360*126) points in the circumferential and radial direction respectively was chosen. Numerous runs have been carried out on the computing cluster at Brunel University. To simulate each case using 8 processors it takes nearly 2000 hours for the results to converge. Figure 3 shows how the results converged for one specific case. The Reynolds number was kept constant at Re=100 for all cases. To initiate the vortex shedding, we applied a random perturbation to the flow. **Figure 4** shows that the results are not dependent on the initial perturbation.

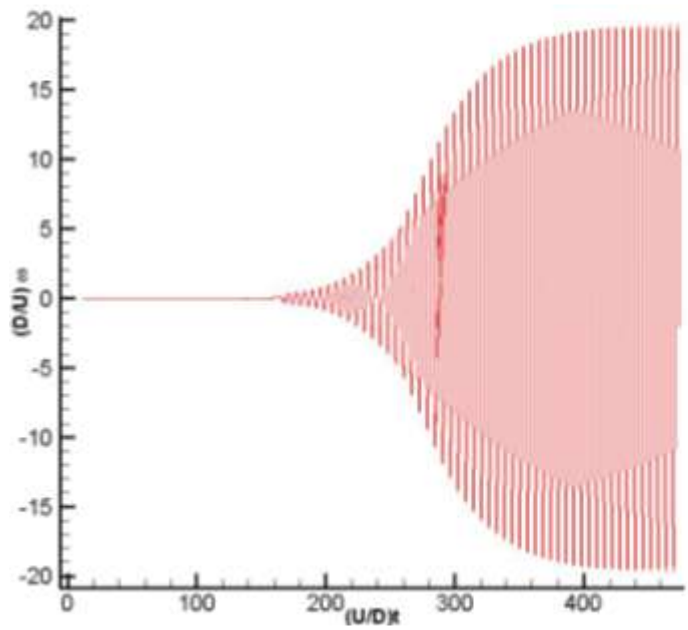


Figure 3: The results of the simulation Converges- Re=100, I=0.333, K=0.3648 Ur=6

In this study, the effect of a rotational degree of freedom on vortex shedding and lock-in phenomena was investigated and the results were compared with flow around a stationary cylinder. Initially an attempt was made to perform a simulation with a rotational d.o.f without any restoring force (K=0). As a result, the cylinder was observed to rotate in only one direction. It was therefore decided to add a restoring force by the introduction of a rotational spring (k>0). To establish which moment of inertia, I, would be relevant to our problem, we assumed a solid cylinder with the same density of water ($1000 kg/m^3$). The diameter of the cylinder was chosen to be 20 cm; as a result I= $(1/8)mD^2=(1/32)\pi\rho D^4 =0.157 kg/m^2$. (For the case with D=0.2m and a density of water equal to 1000 kg/m^3, the equivalent moment of inertia for the cylinder becomes I=0.157 kg.m^2) and the corresponding non-dimensional moment of inertia becomes I=0.5. Figure 5 on the left shows the effect of inertia of the cylinder on the frequency of the vortex shedding of the cylinder (with a constant rotational stiffness k=0.05). For high (low) amounts of inertia the frequency decreases (increases) dramatically. Figure 5, right, depicts the relation between K/I and the natural frequency of the system and the frequency of the rotational velocity (ω). This graph clearly proves equation (2). The power of the K/I is 0.5045, which is almost the same as predicted by theory (0.5) and the coefficient is 0.1598 which is very close to the coefficient $1/2\pi$ =0.1591 in eq. (2).

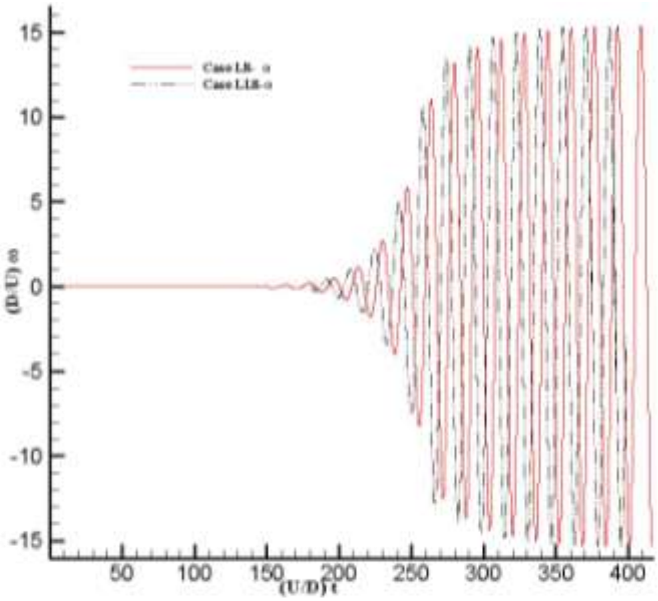


Figure 4: The effect of changes in random perturbation on the convergence of the results for two similar cases-Re=100, I=0.333 K=0.05, Ur=16

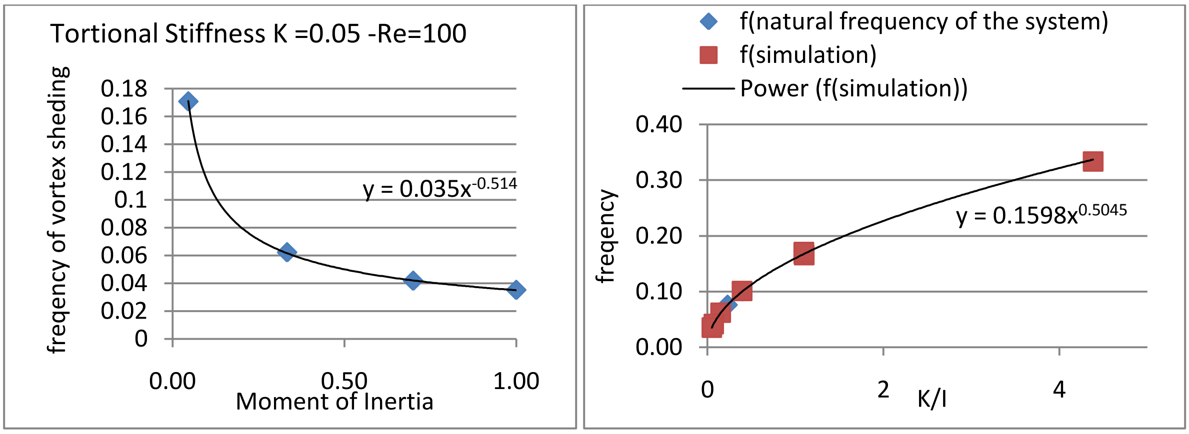


Figure 5 –left: Natural frequency verses inertia when K=0.05 constant. Right: natural frequency and vortex shedding frequency verses (K/I), the parameters are non-dimensional.

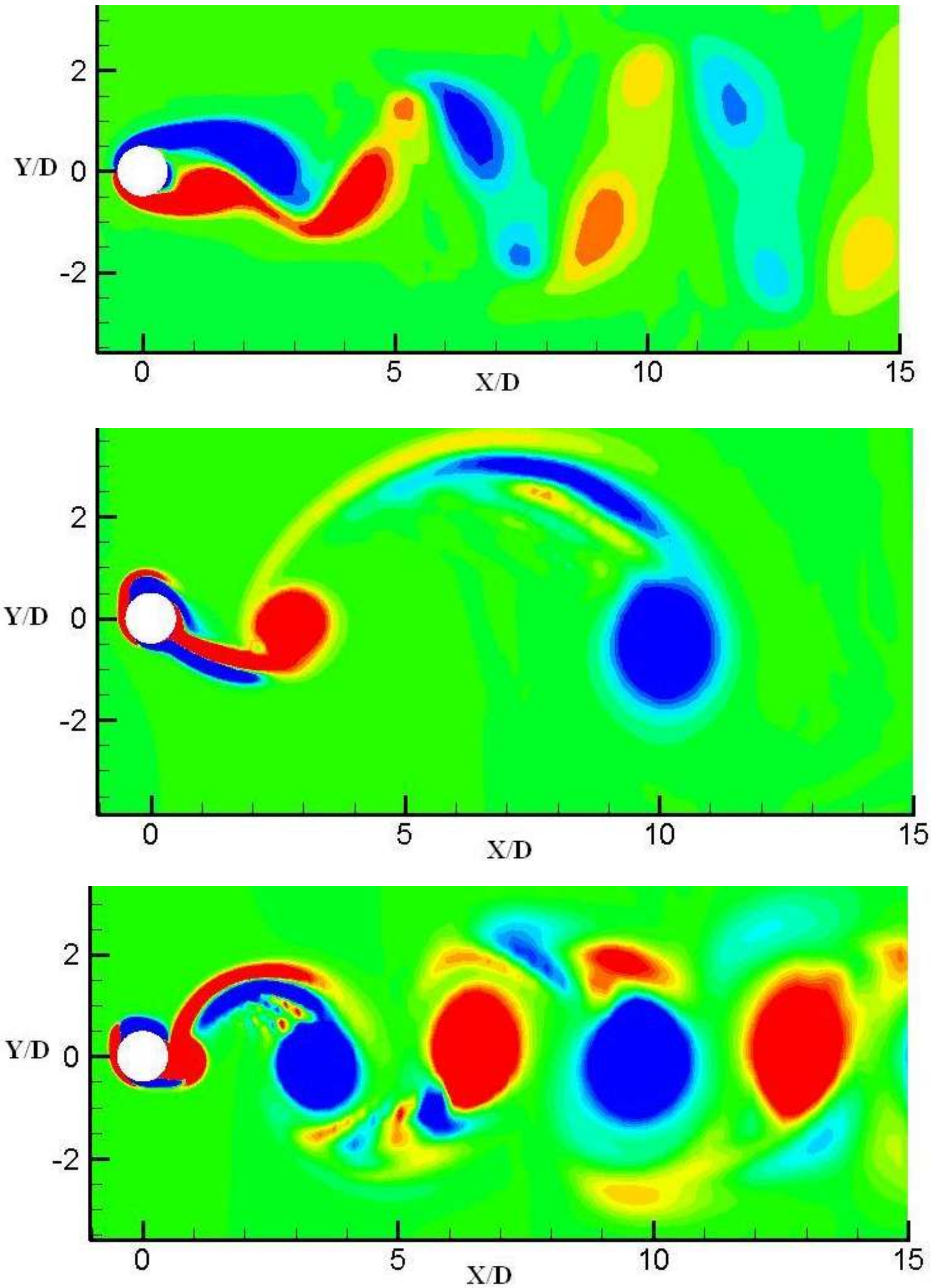


Figure 6 : Vortex Shedding Re=100 - first case: stationary cylinder, St=0.167, Second case I=0.333 and K=0.05, Ur=16, St=0.0625, third case: I =0.333, K= 0.365, Ur=6, St=0.167

Figure 6 shows vortex shedding for 3 different cases in a two dimensional flow at Re=100. The first one is the stationary cylinder; the frequency of the vortex shedding for this case is 0.17. These result exactly match the results of Roshko [7], who measured the frequencies using a hot-wire velocity probe. For the low Reynolds number laminar region Roshko condensed his results to an equation of the form $St = 0.212\ (1 - 21.2\ /\ Re)$ where St is the Strouhal number S=FD/U [7]. The second and third pictures in figure 6 show the vortex shedding from a cylinder with a rotational degree of freedom. For the cases I=0.333 combined with k=0.05 and k=0.365, the frequencies of vortex shedding become f=0.0625 and f=0.167 respectively. In figure 7, the effect of rotational d.o.f was compared with the stationary cylinder. Etienne and Fontaine [5] observed that the introduction of a rotational degree of freedom causes a reduction in the vortex-induced vibration in the transverse direction with the flow [5]. It implies that we should expect a lower lift when we have a rotational d.o.f. in combination with spatial degrees of freedom. In the absence of a spatial degree of freedom, our results show a completely different behaviour and predict a significant increase in unsteady lift forces acting on the cylinder due to the Magnus effect.

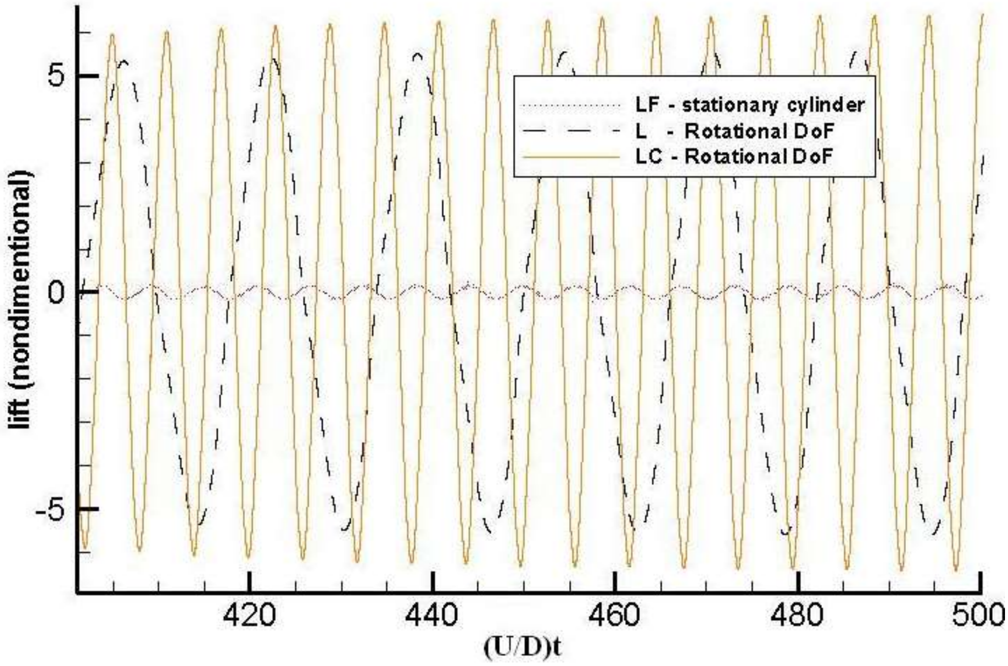


Figure 7- lift for three cases. first case : stationary cylinder, second case: Rotational d.o.f I=0.333 and K=0.05, third case: Rotational degree of freedom I=0.333, K=0.365

4. Conclusion

The introduction of a rotational degree of freedom which allows the cylinder to rotate about its axis, has a significant effect on the pattern of vortex shedding at low Reynolds numbers. In all cases considered, the vortex shedding locks-in to the natural frequency of the inertial/spring system. Compared to the baseline simulation of flow around a stationary cylinder, the addition of a rotational degree of freedom to the cylinder was observed to significantly increase unsteady lift forces because of the Magnus effect, while also the drag forces were not diminished.. In the near future, we aim to complete the present parametric study of the effects of inertia/spring stiffness on the flow pattern and the lift and drag forces.

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